Objectives:

- State the Fundamental Theorem of Calculus.
- Apply the Fundamental Theorem of Calculus to work with functions defined as integrals.

Example 1 If $g(x) = \int_1^x t^3 \, dt$, find a formula for $g(x)$ that doesn't involve an integral and calculate $g'(x)$.

The Fundamental Theorem of Calculus (Part I)

If f is _____, then the function g defined by

$$g(x) =$$

is an antiderivative of f, that is, _____.

Example 2 Find the derivative of the function $g(x) = \int_0^x \sqrt{1+t^2} \, dt$.

The Fundamental Theorem of Calculus

Suppose f is continuous on $[a, b]$.

1. If $g(x) = \displaystyle\int_a^x f(t)\ dt$, then $g'(x) = f(x)$.
2. $\displaystyle\int_a^b f(x)\ dx = F(b) - F(a)$, where F is any antiderivative of f, that is $F' = f$.

For each of the following functions, find the derivative.

1. $g(x) = \displaystyle\int_1^x \frac{1}{t^3 + 1}\ dt$

2. $r(y) = \displaystyle\int_2^y t^2 \sin(t)\ dt$

3. $F(x) = \displaystyle\int_x^\pi \sqrt{1 + \sec(t)}\ dt$

4. $f(x) = \displaystyle\int_1^{x^4} \sec(t)\ dt$

5. $h(x) = \displaystyle\int_{2x}^{3x} \frac{u^2 - 1}{u^2 + 1}\ du$